General Relativity step by step errata

Oct 29 errors fixed

TOC: 2.2. Light should be capitalized.

Page 2. $8\pi\frac{G}{c^4}$ There's a close quote with no corresponding open quote.

Page 5 chain rule should have u^2 differentiated should be $\frac{du}{dx}$ and answer should by $2(x3+1)(3x2)$ corrected $\frac{df}{du}$ to $\frac{df(u)}{du}$

0.1 December 2, 2023

0.1.1 page 5 - original

$\frac{df(x^n)}{dx} = nx^{n-1}$

0.1.2 page 5 - replacement

$\frac{d(x^n)}{dx} = nx^{n-1}$

0.1.3 page 6 - original

$$\frac{\partial f(x(\lambda), y(\lambda))))}{\partial x(\lambda)} = \frac{\partial f(x(\lambda), y(\lambda))}{\partial x(\lambda)} \frac{dx(\lambda)}{d\lambda}$$

0.1.4 page 6 - replacement

$$\frac{\partial f(x(\lambda), y(\lambda))}{\partial x(\lambda)} = \frac{\partial f(x(\lambda), y(\lambda))}{\partial x(\lambda)} \frac{dx(\lambda)}{d\lambda}$$

0.1.5 page 9 original

- A row or column of components (could be scalars or scalar functions). Examples are $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 3t \\ 5t \end{bmatrix}$ or $\begin{bmatrix} \hat{i} & \hat{j} \end{bmatrix}$.
- The count of all components in a vector is the dimension of the vector
- There are special vectors called basis vectors, one for each of the dimensions
- The vector is viewed as a linear combination of components and basis vectors

0.1.6 page 9 - replacement

- A row or column of components (could be scalars or scalar functions). Examples are $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 3t \\ 5t \end{bmatrix}$ or $\begin{bmatrix} 6 & 2 \end{bmatrix}$.

- The count of all components in a vector is the dimension of the vector
- There are special vectors called basis vectors, one for each of the dimensions
- The vector is viewed as a linear combination of components and basis vectors

0.1.7 page 10 - original

$$\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} \hat{i}, \hat{j} \end{bmatrix} = a\hat{i} + b\hat{j}$$

Where a and b are scalar components that could be integers. For example $a = 6, b = 2$:

$$\vec{v} = \begin{bmatrix} 6 \\ 2 \end{bmatrix} \begin{bmatrix} \hat{i}, \hat{j} \end{bmatrix} = 6\hat{i} + 2\hat{j}$$

There are many choices for basis vectors, but choosing different basis vectors (say $\vec{i}$ vs $\hat{i}$) change the components of the same vector. A vector doesn't change (is invariant) even if you describe it with two different sets of basis vectors. Thus you must "adjust" the components. Let $\vec{i} = 2\hat{i}, \vec{j} = \hat{j}$ then:

$$\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} \hat{i}, \hat{j} \end{bmatrix} = a\hat{i} + b\hat{j} = \begin{bmatrix} \frac{1}{2}a \\ b \end{bmatrix} \begin{bmatrix} \vec{i}, \vec{j} \end{bmatrix} = \frac{1}{2}a\vec{i} + b\vec{j}$$

0.1.8 Page 10 - replacement

$$\vec{v} = a\hat{i} + b\hat{j}$$

Where a and b are scalar components that could be integers. For example $a = 6, b = 2$:

$$\vec{v} = 6\hat{i} + 2\hat{j}$$

There are many choices for basis vectors, but choosing different basis vectors (say $\vec{i}$ vs $\hat{i}$) change the components of the same vector. A vector doesn't change (is invariant) even if you describe it with two different sets of basis vectors. Thus you must "adjust" the components. Let $\vec{i} = 2\hat{i}, \vec{j} = \hat{j}$ then:

$$\vec{v} = a\hat{i} + b\hat{j} = \frac{1}{2}a\vec{i} + b\vec{j}$$

0.1.9 Page 14 - orginal

$$\vec{a} \cdot \vec{b} = \vec{a}(\vec{b}^T)$$

Where $(\vec{b})^T$ is the row vector transposed into a column vector. Example of **dot** product:

Let $\vec{a} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, then $\vec{a} \cdot \vec{b} = \vec{a}(\vec{b}^T) = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} =$

0.1.10 Page 14 - replacement

$$\vec{a} \cdot \vec{b} = \vec{a}^T(\vec{b})$$

Where $(\vec{a})^T$ is the row vector transposed into a column vector. Example of **dot** product:

Let $\vec{a} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, then $\vec{a} \cdot \vec{b} = \vec{a}^T(\vec{b}) = \begin{bmatrix} 5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} =$

0.2 June 8, 2025

0.2.1 Page pp187 - original

1. Remember $\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu})$
2. $\Gamma^t_{tt} = \frac{1}{2} g^{t\sigma}(\partial_t g_{\sigma t} + \partial_t g_{\sigma t} - \partial_\sigma g_{tt})$ - only $\sigma = t$ is worth calculating because the metric is a diagonal matrix - all components beside the diagonal ones are zero.
3. $\Gamma^t_{tt} = \frac{1}{2} g^{tt}(\partial_t g_{tt} + \partial_t g_{tt} - \partial_t g_{tt}) = 0$ - because $g_{tt} = -A(m,r)$ does not depend on t. Thus the partial derivative is zero.
4. $\Gamma^t_{tr} = \frac{1}{2} g^{tt}(\partial_r g_{tt} + \partial_t g_{tr} - \partial_t g_{tr}) = \frac{1}{2} \frac{1}{-A(m,r)} \partial_r(-A(m,r))$
5. $\Gamma^t_{rt} = \Gamma^t_{tr} = \frac{1}{2} \frac{1}{-A(m,r)} \partial_r(-A(m,r))$
6. $\Gamma^r_{tt} = \frac{1}{2} g^{rr}(\partial_t g_{rt} + \partial_t g_{rt} - \partial_r g_{tt}) = \frac{1}{2B(m,r)}(-\partial_r(-A(m,r)))$
7. $\Gamma^r_{rr} = \frac{1}{2} g^{rr}(\partial_r g_{rr} + \partial_r g_{rr} - \partial_r g_{rr}) = \frac{1}{2} \frac{1}{B(m,r)}(-\partial_r B(m,r))$
8. $\Gamma^r_{\theta\theta} = \frac{1}{2} g^{rr}(\partial_\theta g_{\theta r} + \partial_\theta g_{\theta r} - \partial_r g_{\theta\theta}) = \frac{1}{2} \frac{1}{B(m,r)}(-2r) = \frac{-r}{B(m,r)}$
9. $\Gamma^r_{\varphi\varphi} = \frac{1}{2} g^{rr}(\partial_\varphi g_{r\varphi} + \partial_\varphi g_{r\varphi} - \partial_r g_{\varphi\varphi}) = \frac{1}{2} \frac{1}{B(m,r)}(-2r \sin^2)\theta = \frac{-r \sin^2 \theta}{B(m,r)}$
10. $\Gamma^\theta_{\varphi\varphi} = \frac{1}{2} g^{\theta\theta}(\partial_\varphi g_{\theta\varphi} + \partial_\varphi g_{\theta\varphi} - \partial_\theta g_{\varphi\varphi}) = \frac{1}{2} \frac{1}{r^2}(-r^2 2 \sin\theta \cos\theta) = -\sin\theta\cos\theta$
11. $\Gamma^\theta_{r\theta} = \frac{1}{2} g^{\theta\theta}(\partial_\theta g_{\theta r} + \partial_r g_{\theta\theta} - \partial_\theta g_{r\theta}) = \frac{1}{2} \frac{1}{r^2} 2r = \frac{1}{r}$
12. $\Gamma^\varphi_{r\varphi} = \frac{1}{2} g^{\varphi\varphi}(\partial_\varphi g_{\varphi r} + \partial_r g_{\varphi\varphi} - \partial_\varphi g_{r\varphi}) = \frac{1}{2} \frac{1}{r^2 \sin^2\theta} 2r \sin^2\theta = \frac{1}{r}$
13. $\Gamma^\varphi_{\theta\varphi} = \frac{1}{2} g^{\varphi\varphi}(\partial_\varphi g_{\varphi\theta} + \partial_\theta g_{\varphi\varphi} - \partial_\varphi g_{\theta\varphi}) = \frac{1}{2} \frac{1}{r^2 \sin^2\theta} r^2 2 \sin\theta\cos\theta = \cot\theta$

The other values of the connection are either zeros or repeats. Thus, we have this table or legend or key for what follows (we've simplified $A = A(m,r)$ and $B = B(m,r)$):

$\Gamma^t_{tr} = \Gamma^t_{rt} = \frac{\partial_r A}{2A}$	$\Gamma^r_{tt} = \frac{\partial_r A}{2B}$	$\Gamma^r_{rr} = -\frac{\partial_r B}{2B}$	$\Gamma^r_{\theta\theta} = \frac{-r}{B}$
$\Gamma^r_{\varphi\varphi} = \frac{-r\sin^2\theta}{B}$	$\Gamma^\varphi_{\theta\varphi} = \Gamma^\varphi_{\varphi\theta} = \cot\theta$	$\Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta$	$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \Gamma^\varphi_{r\varphi} = \Gamma^\varphi_{\varphi r} = \frac{1}{r}$

$R_{\beta\nu} = \partial_\mu \Gamma^\mu_{\beta\nu} - \partial_\nu \Gamma^\mu_{\beta\mu} + \Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{\beta\nu} - \Gamma^\mu_{\sigma\nu}\Gamma^\sigma_{\beta\mu}$ There are 16 combinations (two indices and 4 dimensions = 4x4) to resolve, plus many embedded (Einstein) summations. Rather than crank through them all, we will examine $R_{tt}, R_{rr}, R_{\theta\theta}$.

1. $R_{tt} = \partial_\mu \Gamma^\mu_{tt} - \partial_t \Gamma^\mu_{t\mu} + \Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{tt} - \Gamma^\mu_{\sigma t}\Gamma^\sigma_{t\mu}$
   (a) $\partial_\mu \Gamma^\mu_{tt} = \partial_t \Gamma^t_{tt} + \partial_r \Gamma^r_{tt} + \partial_\theta \Gamma^\theta_{tt} + \partial_\varphi \Gamma^\varphi_{tt}$ first term
   (b) $\partial_\mu \Gamma^\mu_{tt} = \partial_t \cancel{\Gamma}^t_{tt} + \partial_r \left(\frac{\partial_r A}{2B}\right) + \partial_\theta \cancel{\Gamma}^\theta_{tt} + \partial_\varphi \cancel{\Gamma}^\varphi_{tt}$ substitution applied
   (c) $\partial_t \Gamma^\mu_{t\mu} = \partial_t \Gamma^t_{tt} + \partial_t \Gamma^r_{tr} + \partial_t \Gamma^\theta_{t\theta} + \partial_t \Gamma^\varphi_{t\varphi}$ second term
   (d) $\partial_t \Gamma^\mu_{t\mu} = \partial_t \cancel{\Gamma}^t_{tt} + \partial_t \cancel{\Gamma}^r_{tr} + \partial_t \cancel{\Gamma}^\theta_{t\theta} + \partial_t \cancel{\Gamma}^\varphi_{t\varphi}$ substitution applied
   (e) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{tt} = \Gamma^t_{tt}\Gamma^t_{tt} + \Gamma^r_{tr}\Gamma^t_{tt} + \Gamma^\theta_{t\theta}\Gamma^t_{tt} + \Gamma^\varphi_{t\varphi}\Gamma^t_{tt}$
   $+\Gamma^t_{rt}\Gamma^r_{tt} + \Gamma^r_{rr}\Gamma^r_{tt} + \Gamma^\theta_{r\theta}\Gamma^r_{tt} + \Gamma^\varphi_{r\varphi}\Gamma^r_{tt}$
   $+\Gamma^t_{\theta t}\Gamma^\theta_{tt} + \Gamma^r_{\theta r}\Gamma^\theta_{tt} + \Gamma^\theta_{\theta\theta}\Gamma^\theta_{tt} + \Gamma^\varphi_{\theta\varphi}\Gamma^\theta_{tt}$
   $+\Gamma^t_{\varphi t}\Gamma^\varphi_{tt} + \Gamma^r_{\varphi r}\Gamma^\varphi_{tt} + \Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{tt} + \Gamma^\varphi_{\varphi\varphi}\Gamma^\varphi_{tt}$
   (f) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{tt} = \cancel{\Gamma}^t_{tt}\cancel{\Gamma}^t_{tt} + \cancel{\Gamma}^r_{tr}\cancel{\Gamma}^t_{tt} + \cancel{\Gamma}^\theta_{t\theta}\cancel{\Gamma}^t_{tt} + \cancel{\Gamma}^\varphi_{t\varphi}\cancel{\Gamma}^t_{tt}$
   $+\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right) + \left(-\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r A}{2B}\right) + \left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right) + \left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right)$
   $+\cancel{\Gamma}^t_{\theta t}\cancel{\Gamma}^\theta_{tt} + \cancel{\Gamma}^r_{\theta r}\cancel{\Gamma}^\theta_{tt} + \cancel{\Gamma}^\theta_{\theta\theta}\cancel{\Gamma}^\theta_{tt} + (\cot\theta)\cancel{\Gamma}^\theta_{tt}$
   $+\cancel{\Gamma}^t_{\varphi t}\cancel{\Gamma}^\varphi_{tt} + \cancel{\Gamma}^r_{\varphi r}\cancel{\Gamma}^\varphi_{tt} + \cancel{\Gamma}^\theta_{\varphi\theta}\cancel{\Gamma}^\varphi_{tt} + \cancel{\Gamma}^\varphi_{\varphi\varphi}\cancel{\Gamma}^\varphi_{tt}$ substitution applied
   (g) $\Gamma^\mu_{\sigma t}\Gamma^\sigma_{t\mu} = \Gamma^t_{tt}\Gamma^t_{tt} + \Gamma^r_{tt}\Gamma^t_{tr} + \Gamma^\theta_{tt}\Gamma^t_{t\theta} + \Gamma^\varphi_{tt}\Gamma^t_{t\varphi}$
   $+\Gamma^t_{rt}\Gamma^r_{tt} + \Gamma^r_{rt}\Gamma^r_{tr} + \Gamma^\theta_{rt}\Gamma^r_{t\theta} + \Gamma^\varphi_{rt}\Gamma^r_{t\varphi}$
   $+\Gamma^t_{\theta t}\Gamma^\theta_{tt} + \Gamma^r_{\theta t}\Gamma^\theta_{tr} + \Gamma^\theta_{\theta t}\Gamma^\theta_{t\theta} + \Gamma^\varphi_{\theta t}\Gamma^\theta_{t\varphi}$
   $+\Gamma^t_{\varphi t}\Gamma^\varphi_{tt} + \Gamma^r_{\varphi t}\Gamma^\varphi_{tr} + \Gamma^\theta_{\varphi t}\Gamma^\varphi_{t\theta} + \Gamma^\varphi_{\varphi t}\Gamma^\varphi_{t\varphi}$
   (h) $\Gamma^\mu_{\sigma t}\Gamma^\sigma_{t\mu} = \cancel{\Gamma}^t_{tt}\cancel{\Gamma}^t_{tt} + \left(\frac{\partial_r A}{2B}\right)\left(\frac{\partial_r A}{2A}\right) + \cancel{\Gamma}^\theta_{tt}\cancel{\Gamma}^t_{t\theta} + \cancel{\Gamma}^\varphi_{tt}\cancel{\Gamma}^t_{t\varphi}$
   $+\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right) + \cancel{\Gamma}^r_{rt}\cancel{\Gamma}^r_{tr} + \cancel{\Gamma}^\theta_{rt}\cancel{\Gamma}^r_{t\theta} + \cancel{\Gamma}^\varphi_{rt}\cancel{\Gamma}^r_{t\varphi}$
   $+\cancel{\Gamma}^t_{\theta t}\cancel{\Gamma}^\theta_{tt} + \cancel{\Gamma}^r_{\theta t}\cancel{\Gamma}^\theta_{tr} + \cancel{\Gamma}^\theta_{\theta t}\cancel{\Gamma}^\theta_{t\theta} + \cancel{\Gamma}^\varphi_{\theta t}\cancel{\Gamma}^\theta_{t\varphi}$
   $+\cancel{\Gamma}^t_{\varphi t}\cancel{\Gamma}^\varphi_{tt} + \cancel{\Gamma}^r_{\varphi t}\cancel{\Gamma}^\varphi_{tr} + \cancel{\Gamma}^\theta_{\varphi t}\cancel{\Gamma}^\varphi_{t\theta} + \cancel{\Gamma}^\varphi_{\varphi t}\cancel{\Gamma}^\varphi_{t\varphi}$ substitution applied
   (i)
   $$R_{tt} = \partial_r \left(\frac{\partial_r A}{2B}\right) + \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right) + \left(-\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r A}{2B}\right) + \left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right) + \left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right) - \left[\left(\frac{\partial_r A}{2B}\right)\left(\frac{\partial_r A}{2A}\right) + \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right)\right] \tag{1}$$
   (j) $R_{tt} = \partial_r \frac{\partial_r A}{2B} + \left[\frac{\partial_r A}{2A}\frac{\partial_r A}{2B} - \frac{\partial_r B}{2B}\frac{\partial_r A}{2B} + \frac{\partial_r A}{2B}\frac{1}{r} + \frac{\partial_r A}{2B}\frac{1}{r}\right] - \left[\frac{\partial_r A}{2B}\left(\frac{\partial_r A}{2A}\right) - \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right)\right]$
   (k) $R_{tt} = \partial_r \frac{\partial_r A}{2B} - \frac{\partial_r B}{2B}\frac{\partial_r A}{2B} + 2\frac{\partial_r A}{2B}\frac{1}{r} + \frac{\partial_r A}{2B}\frac{\partial_r A}{2A}$
   (l) $R_{tt} = \frac{1}{2B}\partial_r'' A - \frac{\partial_r A \partial_r B}{4B^2} + \frac{\partial_r B \partial_r A}{4B} + \frac{\partial_r A}{Br} + \frac{(\partial_r A)^2}{4AB}$
   (m)
   $$R_{tt} = 2ABr\partial_r'' A - Ar\partial_r A\partial_r B + ABr\partial_r A\partial_r B + 4BA\partial_r A + Br(\partial_r A)^2 \tag{2}$$
2. $R_{rr} = \partial_\mu \Gamma^\mu_{rr} - \partial_r \Gamma^\mu_{r\mu} + \Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{rr} - \Gamma^\mu_{\sigma r}\Gamma^\sigma_{r\mu}$
   (a) $\partial_\mu \Gamma^\mu_{rr} = \partial_t \Gamma^t_{rr} + \partial_r \Gamma^r_{rr} + \partial_\theta \Gamma^\theta_{rr} + \partial_\varphi \Gamma^\varphi_{rr}$

(b) $\partial_\mu \Gamma^\mu_{rr} \partial_t \cancel{\Gamma}^t_{rr} + \partial_r \left(-\frac{\partial_r B}{2B}\right) + \partial_\theta \cancel{\Gamma}^\theta_{rr} + \partial_\varphi \cancel{\Gamma}^\varphi_{rr}$ substitution applied

(c) $\partial_r \Gamma^\mu_{r\mu} = \partial_r \Gamma^t_{rt} + \partial_r \Gamma^r_{rr} + \partial_r \Gamma^\theta_{r\theta} + \partial_r \Gamma^\varphi_{r\varphi}$

(d) $\partial_r \Gamma^\mu_{r\mu} = \partial_r \left(\frac{\partial_r A}{2A}\right) + \partial_r \left(-\frac{\partial_r B}{2B}\right) + \partial_r \left(\frac{1}{r}\right) + \partial_r \left(\frac{1}{r}\right)$ substitution applied

(e) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{rr} = \Gamma^t_{tt}\Gamma^t_{rr} + \Gamma^r_{tr}\Gamma^t_{rr} + \Gamma^\theta_{t\theta}\Gamma^t_{rr} + \Gamma^\varphi_{t\varphi}\Gamma^t_{rr}$
$+\Gamma^t_{rt}\Gamma^r_{rr} + \Gamma^r_{rr}\Gamma^r_{rr} + \Gamma^\theta_{r\theta}\Gamma^r_{rr} + \Gamma^\varphi_{r\varphi}\Gamma^r_{rr}$
$+\Gamma^t_{\theta t}\Gamma^\theta_{rr} + \Gamma^r_{\theta r}\Gamma^\theta_{rr} + \Gamma^\theta_{\theta\theta}\Gamma^\theta_{rr} + \Gamma^\varphi_{\theta\varphi}\Gamma^\theta_{rr}$
$+\Gamma^t_{\varphi t}\Gamma^\varphi_{rr} + \Gamma^r_{\varphi r}\Gamma^\varphi_{rr} + \Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{rr} + \Gamma^\varphi_{\varphi\varphi}\Gamma^\varphi_{rr}$

(f) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{rr} = \cancel{\Gamma}^t_{tt}\cancel{\Gamma}^t_{rr} + \cancel{\Gamma}^r_{tr}\cancel{\Gamma}^t_{rr} + \cancel{\Gamma}^\theta_{t\theta}\cancel{\Gamma}^t_{rr} + \cancel{\Gamma}^\varphi_{t\varphi}\cancel{\Gamma}^t_{rr}$
$+\left(\frac{\partial_r A}{2A}\right)\left(-\frac{\partial_r B}{2B}\right) + \left(-\frac{\partial_r B}{2B}\right)\left(-\frac{\partial_r B}{2B}\right) + \left(\frac{1}{r}\right)\left(-\frac{\partial_r B}{2B}\right) + \left(\frac{1}{r}\right)\left(-\frac{\partial_r B}{2B}\right)$
$+\cancel{\Gamma}^t_{\theta t}\cancel{\Gamma}^\theta_{rr} + \cancel{\Gamma}^r_{\theta r}\cancel{\Gamma}^\theta_{rr} + \cancel{\Gamma}^\theta_{\theta\theta}\cancel{\Gamma}^\theta_{rr} + (\cot\theta)\cancel{\Gamma}^\theta_{rr}$
$+\cancel{\Gamma}^t_{\varphi t}\cancel{\Gamma}^\varphi_{rr} + \cancel{\Gamma}^r_{\varphi r}\cancel{\Gamma}^\varphi_{rr} + \cancel{\Gamma}^\theta_{\varphi\theta}\cancel{\Gamma}^\varphi_{rr} + \cancel{\Gamma}^\varphi_{\varphi\varphi}\cancel{\Gamma}^\varphi_{rr}$ substitution applied

(g) $\Gamma^\mu_{\sigma r}\Gamma^\sigma_{r\mu} = \Gamma^t_{tr}\Gamma^t_{rt} + \Gamma^r_{tr}\Gamma^t_{rr} + \Gamma^\theta_{tr}\Gamma^t_{r\theta} + \Gamma^\varphi_{tr}\Gamma^t_{r\varphi}$
$+\Gamma^t_{rr}\Gamma^r_{rt} + \Gamma^r_{rr}\Gamma^r_{rr} + \Gamma^\theta_{rr}\Gamma^r_{r\theta} + \Gamma^\varphi_{rr}\Gamma^r_{r\varphi}$
$+\Gamma^t_{\theta r}\Gamma^\theta_{rt} + \Gamma^r_{\theta r}\Gamma^\theta_{rr} + \Gamma^\theta_{\theta r}\Gamma^\theta_{r\theta} + \Gamma^\varphi_{\theta r}\Gamma^\theta_{r\varphi}$
$+\Gamma^t_{\varphi r}\Gamma^\varphi_{rt} + \Gamma^r_{\varphi r}\Gamma^\varphi_{rr} + \Gamma^\theta_{\varphi r}\Gamma^\varphi_{r\theta} + \Gamma^\varphi_{\varphi r}\Gamma^\varphi_{r\varphi}$

(h) $\Gamma^\mu_{\sigma r}\Gamma^\sigma_{r\mu} = \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2A}\right) + \cancel{\Gamma}^r_{tr}\cancel{\Gamma}^t_{rr} + \cancel{\Gamma}^\theta_{tr}\cancel{\Gamma}^t_{r\theta} + \cancel{\Gamma}^\varphi_{tr}\cancel{\Gamma}^t_{r\varphi}$
$+\cancel{\Gamma}^t_{rr}\cancel{\Gamma}^r_{rt} + \left(-\frac{\partial_r B}{2B}\right)\left(-\frac{\partial_r B}{2B}\right) + \cancel{\Gamma}^\theta_{rr}\cancel{\Gamma}^r_{r\theta} + \cancel{\Gamma}^\varphi_{rr}\cancel{\Gamma}^r_{r\varphi}$
$+\cancel{\Gamma}^t_{\theta r}\cancel{\Gamma}^\theta_{rt} + \cancel{\Gamma}^r_{\theta r}\cancel{\Gamma}^\theta_{rr} + \left(\frac{1}{r}\right)\left(\frac{1}{r}\right) + \cancel{\Gamma}^\varphi_{\theta r}\cancel{\Gamma}^\theta_{r\varphi}$
$+\cancel{\Gamma}^t_{\varphi r}\cancel{\Gamma}^\varphi_{rt} + \cancel{\Gamma}^r_{\varphi r}\cancel{\Gamma}^\varphi_{rr} + \cancel{\Gamma}^\theta_{\varphi r}\cancel{\Gamma}^\varphi_{r\theta} + \left(\frac{1}{r}\right)\left(\frac{1}{r}\right)$ substitution applied

(i)

$$\begin{aligned} R_{rr} = \partial_r \left(-\frac{\partial_r B}{2B}\right) - \\ \left[\partial_r \left(\frac{\partial_r A}{2A}\right) + \partial_r \left(-\frac{\partial_r B}{2B}\right) + \partial_r \left(\frac{1}{r}\right) + \partial_r \left(\frac{1}{r}\right)\right] + \\ \left(\frac{\partial_r A}{2A}\right)\left(-\frac{\partial_r B}{2B}\right) + \left(-\frac{\partial_r B}{2B}\right)\left(-\frac{\partial_r B}{2B}\right) + \left(\frac{1}{r}\right)\left(-\frac{\partial_r B}{2B}\right) + \left(\frac{1}{r}\right)\left(-\frac{\partial_r B}{2B}\right) - \\ \left[\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2A}\right) + \left(-\frac{\partial_r B}{2B}\right)\left(-\frac{\partial_r B}{2B}\right) + \left(\frac{1}{r}\right)\left(\frac{1}{r}\right) + \left(\frac{1}{r}\right)\left(\frac{1}{r}\right)\right] \end{aligned} \tag{3}$$

(j) $R_{rr} = \cancel{\partial_r(-\frac{\partial_r B}{2B})} - [\partial_r(\frac{\partial_r A}{2A}) + \cancel{\partial_r(-\frac{\partial_r B}{2B})} + \partial_r\cancel{(\frac{1}{r}) + \partial_r(\frac{1}{r})}] + (\frac{\partial_r A}{2A})(-\frac{\partial_r B}{2B}) + (-\frac{\partial_r B}{2B})(-\frac{\partial_r B}{2B}) +$
$(\frac{1}{r})(-\frac{\partial_r B}{2B}) + (\frac{1}{r})(-\frac{\partial_r B}{2B}) - [(\frac{\partial_r A}{2A})(\frac{\partial_r A}{2A}) + (-\frac{\partial_r B}{2B})(-\frac{\partial_r B}{2B}) + \cancel{(\frac{1}{r})(\frac{1}{r}) + (\frac{1}{r})(\frac{1}{r})}] =$

(k) $R_{rr} = -\frac{\partial_r'' A}{2A} + \frac{\partial_r A \partial_r B}{4AB} - \frac{\partial_r B}{rB} - \frac{(\partial_r A)^2}{4A^2} =$

(l)

$$R_{rr} = -2ArB\partial_r'' A - rB(\partial_r A)^2 + rA\partial_r A\partial_r B - 4A^2\partial_r B \tag{4}$$

3. $R_{\theta\theta} = \partial_\mu \Gamma^\mu_{\theta\theta} - \partial_\theta \Gamma^\mu_{\theta\mu} + \Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{\theta\theta} - \Gamma^\mu_{\sigma\theta}\Gamma^\sigma_{\theta\mu}$

(a) $\partial_\mu \Gamma^\mu_{\theta\theta} = \partial_t \Gamma^t_{\theta\theta} + \partial_r \Gamma^r_{\theta\theta} + \partial_\theta \Gamma^\theta_{\theta\theta} + \partial_\varphi \Gamma^\varphi_{\theta\theta}$

(b) $\partial_\mu \Gamma^\mu_{\theta\theta} = \partial_t \cancel{\Gamma}^t_{\theta\theta} + \partial_r \left(\frac{-r}{B}\right) + \partial_\theta \cancel{\Gamma}^\theta_{\theta\theta} + \partial_\varphi \cancel{\Gamma}^\varphi_{\theta\theta}$ substitution applied

(c) $\partial_\theta \Gamma^\mu_{\theta\mu} = \partial_\theta \Gamma^t_{\theta t} + \partial_\theta \Gamma^r_{\theta r} + \partial_\theta \Gamma^\theta_{\theta\theta} + \partial_\theta \Gamma^\varphi_{\theta\varphi}$

(d) $\partial_\theta \Gamma^\mu_{\theta\mu} = \partial_\theta\cancel{\Gamma}^t_{\theta t}+\partial_\theta\cancel{\Gamma}^r_{\theta r}+\partial_\theta\cancel{\Gamma}^\theta_{\theta\theta}+\partial_\theta\,(\cot\theta)$ substitution applied

(e) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{\theta\theta} = \Gamma^t_{tt}\Gamma^t_{\theta\theta}+\Gamma^r_{tr}\Gamma^t_{\theta\theta}+\Gamma^\theta_{t\theta}\Gamma^t_{\theta\theta}+\Gamma^\varphi_{t\varphi}\Gamma^t_{\theta\theta}$
$+\Gamma^t_{rt}\Gamma^r_{\theta\theta}+\Gamma^r_{rr}\Gamma^r_{\theta\theta}+\Gamma^\theta_{r\theta}\Gamma^r_{\theta\theta}+\Gamma^\varphi_{r\varphi}\Gamma^r_{\theta\theta}$
$+\Gamma^t_{\theta t}\Gamma^\theta_{\theta\theta}+\Gamma^r_{\theta r}\Gamma^\theta_{\theta\theta}+\Gamma^\theta_{\theta\theta}\Gamma^\theta_{\theta\theta}+\Gamma^\varphi_{\theta\varphi}\Gamma^\theta_{\theta\theta}$
$+\Gamma^t_{\varphi t}\Gamma^\varphi_{\theta\theta}+\Gamma^r_{\varphi r}\Gamma^\varphi_{\theta\theta}+\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{\theta\theta}+\Gamma^\varphi_{\varphi\varphi}\Gamma^\varphi_{\theta\theta}$

(f) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{\theta\theta} = \cancel{\Gamma}^t_{tt}\cancel{\Gamma}^t_{\theta\theta}+\cancel{\Gamma}^r_{tr}\cancel{\Gamma}^t_{\theta\theta}+\cancel{\Gamma}^\theta_{t\theta}\cancel{\Gamma}^t_{\theta\theta}+\cancel{\Gamma}^\varphi_{t\varphi}\cancel{\Gamma}^t_{\theta\theta}$
$+\left(\frac{\partial_r A}{2A}\right)\left(\frac{-r}{B}\right)+\left(-\frac{\partial_r B}{2B}\right)\left(\frac{-r}{B}\right)+\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)+\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)$
$+\cancel{\Gamma}^t_{\theta t}\cancel{\Gamma}^\theta_{\theta\theta}+\cancel{\Gamma}^r_{\theta r}\cancel{\Gamma}^\theta_{\theta\theta}+\cancel{\Gamma}^\theta_{\theta\theta}\cancel{\Gamma}^\theta_{\theta\theta}+(\cot\theta)\,\cancel{\Gamma}^\theta_{\theta\theta}$
$+\cancel{\Gamma}^t_{\varphi t}\cancel{\Gamma}^\varphi_{\theta\theta}+\cancel{\Gamma}^r_{\varphi r}\cancel{\Gamma}^\varphi_{\theta\theta}+\cancel{\Gamma}^\theta_{\varphi\theta}\cancel{\Gamma}^\varphi_{\theta\theta}+\cancel{\Gamma}^\varphi_{\varphi\varphi}\cancel{\Gamma}^\varphi_{\theta\theta}$ substitution applied

(g) $\Gamma^\mu_{\sigma\theta}\Gamma^\sigma_{\theta\mu} = \Gamma^t_{t\theta}\Gamma^t_{\theta t}+\Gamma^r_{t\theta}\Gamma^t_{\theta r}+\Gamma^\theta_{t\theta}\Gamma^t_{\theta\theta}+\Gamma^\varphi_{t\theta}\Gamma^t_{\theta\varphi}$
$+\Gamma^t_{r\theta}\Gamma^r_{\theta t}+\Gamma^r_{r\theta}\Gamma^r_{\theta r}+\Gamma^\theta_{r\theta}\Gamma^r_{\theta\theta}+\Gamma^\varphi_{r\theta}\Gamma^r_{\theta\varphi}$
$+\Gamma^t_{\theta\theta}\Gamma^\theta_{\theta t}+\Gamma^r_{\theta\theta}\Gamma^\theta_{\theta r}+\Gamma^\theta_{\theta\theta}\Gamma^\theta_{\theta\theta}+\Gamma^\varphi_{\theta\theta}\Gamma^\theta_{\theta\varphi}$
$+\Gamma^t_{\varphi\theta}\Gamma^\varphi_{\theta t}+\Gamma^r_{\varphi\theta}\Gamma^\varphi_{\theta r}+\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{\theta\theta}+\Gamma^\varphi_{\varphi\theta}\Gamma^\varphi_{\theta\varphi}$

(h) $\Gamma^\mu_{\sigma\theta}\Gamma^\sigma_{\theta\mu} = \cancel{\Gamma}^t_{t\theta}\cancel{\Gamma}^t_{\theta t}+\cancel{\Gamma}^r_{t\theta}\cancel{\Gamma}^t_{\theta r}+\cancel{\Gamma}^\theta_{t\theta}\cancel{\Gamma}^t_{\theta\theta}+\cancel{\Gamma}^\varphi_{t\theta}\cancel{\Gamma}^t_{\theta\varphi}$
$+\cancel{\Gamma}^t_{r\theta}\cancel{\Gamma}^r_{\theta t}+\cancel{\Gamma}^r_{r\theta}\cancel{\Gamma}^r_{\theta r}+\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)+\cancel{\Gamma}^\varphi_{r\theta}\cancel{\Gamma}^r_{\theta\varphi}$
$+\cancel{\Gamma}^t_{\theta\theta}\cancel{\Gamma}^\theta_{\theta t}+\left(\frac{-r}{B}\right)\left(\frac{1}{r}\right)+\cancel{\Gamma}^\theta_{\theta\theta}\cancel{\Gamma}^\theta_{\theta\theta}+\cancel{\Gamma}^\varphi_{\theta\theta}\cancel{\Gamma}^\theta_{\theta\varphi}$
$+\cancel{\Gamma}^t_{\varphi\theta}\cancel{\Gamma}^\varphi_{\theta t}+\cancel{\Gamma}^r_{\varphi\theta}\cancel{\Gamma}^\varphi_{\theta r}+\cancel{\Gamma}^\theta_{\varphi\theta}\cancel{\Gamma}^\varphi_{\theta\theta}+(\cot\theta)\,(\cot\theta)$ substitution applied

$$\begin{aligned} R_{\theta\theta} = \partial_r\left(\frac{-r}{B}\right)-\\ \partial_\theta\,(\cot\theta)+\\ \left[\left(\frac{\partial_r A}{2A}\right)\left(\frac{-r}{B}\right)+\left(-\frac{\partial_r B}{2B}\right)\left(\frac{-r}{B}\right)+\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)\right]-\\ \left[\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)+\left(\frac{-r}{B}\right)\left(\frac{1}{r}\right)+(\cot\theta)\,(\cot\theta)\right] \end{aligned} \tag{5}$$

(i) $R_{\theta\theta} = \partial_r\frac{-r}{B}-\partial_\theta\cot\theta+[(\frac{\partial_r A}{2A})(\frac{-r}{B})+(-\frac{\partial_r B}{2B})(\frac{-r}{B})+(\frac{1}{r})(\frac{-r}{B})+(\frac{1}{r})(\frac{-r}{B})]-[(\frac{1}{r})(\frac{-r}{B})+(\frac{-r}{B})(\frac{1}{r})+(\cot\theta)(\cot\theta)]$

i. $\partial_r\cot\theta = -\frac{1}{sin^2\theta}$

ii. $\cot\theta\cot\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{1-sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}-1$

iii. $\partial_r\frac{-r}{B} = \frac{-1}{B}+r\frac{-1}{B^2}\partial_r B$

(j)

$$R_{\theta\theta} = \frac{-1}{B}+1+\frac{r\partial_r A}{2AB}-\frac{r\partial_r B}{2B^2} = -2AB+2AB^2-r\partial_r AB+rA\partial_r B \tag{6}$$

4. Since $R_{tt}=R_{rr}=R_{\theta\theta}=0$ we can solve this system of equations by adding the first eq 8 and eq 10 and dividing by $4A$. We are left with $B\partial_r A + A\partial_r B = 0$

5. $B\partial_r A + A\partial_r B = \partial_r(AB) = 0$ which implies $AB = K$ where $K = const.$

0.2.2 Page pp187 - replacement

1. Remember $\Gamma^{\lambda}_{\mu\nu}{=}\frac{1}{2}g^{\lambda\sigma}(\partial_\nu g_{\sigma\mu}+\partial_\mu g_{\sigma\nu}-\partial_\sigma g_{\mu\nu})$
2. $\Gamma^t_{tt}=\frac{1}{2}g^{t\sigma}(\partial_t g_{\sigma t}+\partial_t g_{\sigma t}-\partial_\sigma g_{tt})$ - only $\sigma=t$ is worth calculating because the metric is a diagonal matrix - all components beside the diagonal ones are zero.
3. $\Gamma^t_{tt}=\frac{1}{2}g^{tt}(\partial_t g_{tt}+\partial_t g_{tt}-\partial_t g_{tt})=0$ - because $g_{tt}=-A(m,r)$ does not depend on t. Thus the partial derivative is zero.
4. $\Gamma^t_{tr}=\frac{1}{2}g^{tt}(\partial_r g_{tt}+\partial_t g_{tr}-\partial_t g_{tr})=\frac{1}{2}\frac{1}{-A(m,r)}\partial_r(-A(m,r))$
5. $\Gamma^t_{rt}=\Gamma^t_{tr}=\frac{1}{2}\frac{1}{-A(m,r)}\partial_r(-A(m,r))$
6. $\Gamma^r_{tt}{=}\frac{1}{2}g^{rr}(\partial_t g_{rt}+\partial_t g_{rt}-\partial_r g_{tt}){=}\frac{1}{2B(m,r)}(-\partial_r(-A(m,r)))$
7. $\Gamma^r_{rr}{=}\frac{1}{2}g^{rr}(\partial_r g_{rr}+\partial_r g_{rr}-\partial_r g_{rr}){=}\frac{1}{2}\frac{1}{B(m,r)}(\partial_r B(m,r))$
8. $\Gamma^r_{\theta\theta}{=}\frac{1}{2}g^{rr}(\partial_\theta g_{\theta r}+\partial_\theta g_{\theta r}-\partial_r g_{\theta\theta}){=}\frac{1}{2}\frac{1}{B(m,r)}(-2r)=\frac{-r}{B(m,r)}$
9. $\Gamma^r_{\varphi\varphi}{=}\frac{1}{2}g^{rr}(\partial_\varphi g_{r\varphi}+\partial_\varphi g_{r\varphi}-\partial_r g_{\varphi\varphi}){=}\frac{1}{2}\frac{1}{B(m,r)}(-2r\sin^2)\theta=\frac{-r\sin^2\theta}{B(m,r)}$
10. $\Gamma^\theta_{\varphi\varphi}{=}\frac{1}{2}g^{\theta\theta}(\partial_\varphi g_{\theta\varphi}+\partial_\varphi g_{\theta\varphi}-\partial_\theta g_{\varphi\varphi})=\frac{1}{2}\frac{1}{r^2}(-r^2 2\sin\theta\cos\theta){=}-\sin\theta\cos\theta$
11. $\Gamma^\theta_{r\theta}{=}\frac{1}{2}g^{\theta\theta}(\partial_\theta g_{\theta r}+\partial_r g_{\theta\theta}-\partial_\theta g_{r\theta}){=}\frac{1}{2}\frac{1}{r^2}2r=\frac{1}{r}$
12. $\Gamma^\varphi_{r\varphi}{=}\frac{1}{2}g^{\varphi\varphi}(\partial_\varphi g_{\varphi r}+\partial_r g_{\varphi\varphi}-\partial_\varphi g_{r\varphi})=\frac{1}{2}\frac{1}{r^2\sin^2\theta}2r\sin^2\theta{=}\frac{1}{r}$
13. $\Gamma^\varphi_{\theta\varphi}{=}\frac{1}{2}g^{\varphi\varphi}(\partial_\varphi g_{\varphi\theta}+\partial_\theta g_{\varphi\varphi}-\partial_\varphi g_{\theta\varphi})=\frac{1}{2}\frac{1}{r^2\sin^2\theta}r^2 2\sin\theta\cos\theta{=}\cot\theta$

The other values of the connection are either zeros or repeats. Thus, we have this table or legend or key for what follows (we've simplified $A=A(m,r)$ and $B=B(m,r)$):

$\Gamma^t_{tr}=\Gamma^t_{rt}=\frac{\partial_r A}{2A}$	$\Gamma^r_{tt}=\frac{\partial_r A}{2B}$	$\Gamma^r_{rr}=\frac{\partial_r B}{2B}$	$\Gamma^r_{\theta\theta}=\frac{-r}{B}$
$\Gamma^r_{\varphi\varphi}=\frac{-r\sin^2\theta}{B}$	$\Gamma^\varphi_{\theta\varphi}=\Gamma^\varphi_{\varphi\theta}=\cot\theta$	$\Gamma^\theta_{\varphi\varphi}=-\sin\theta\cos\theta$	$\Gamma^\theta_{r\theta}=\Gamma^\theta_{\theta r}=\Gamma^\varphi_{r\varphi}=\Gamma^\varphi_{\varphi r}=\frac{1}{r}$

$R_{\beta\nu}=\partial_\mu\Gamma^\mu_{\beta\nu}-\partial_\nu\Gamma^\mu_{\beta\mu}+\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{\beta\nu}-\Gamma^\mu_{\sigma\nu}\Gamma^\sigma_{\beta\mu}$ There are 16 combinations (two indices and 4 dimensions = 4x4) to resolve, plus many embedded (Einstein) summations. Rather than crank through them all, we will examine $R_{tt}, R_{rr}, R_{\theta\theta}$.

1. $R_{tt}=\partial_\mu\Gamma^\mu_{tt}-\partial_t\Gamma^\mu_{t\mu}+\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{tt}-\Gamma^\mu_{\sigma t}\Gamma^\sigma_{t\mu}$
   (a) $\partial_\mu\Gamma^\mu_{tt}=\partial_t\Gamma^t_{tt}{+}\partial_r\Gamma^r_{tt}{+}\partial_\theta\Gamma^\theta_{tt}{+}\partial_\varphi\Gamma^\varphi_{tt}$ first term
   (b) $\partial_\mu\Gamma^\mu_{tt}=\partial_t\cancel{\Gamma^t_{tt}}{+}\partial_r\left(\frac{\partial_r A}{2B}\right){+}\partial_\theta\cancel{\Gamma^\theta_{tt}}{+}\partial_\varphi\cancel{\Gamma^\varphi_{tt}}$ substitution applied
   (c) $\partial_t\Gamma^\mu_{t\mu}=\partial_t\Gamma^t_{tt}{+}\partial_t\Gamma^r_{tr}{+}\partial_t\Gamma^\theta_{t\theta}{+}\partial_t\Gamma^\varphi_{t\varphi}$ second term

(d) $\partial_t\Gamma^\mu_{t\mu} = \partial_t\cancel{\Gamma}^t_{tt}+\partial_t\cancel{\Gamma}^r_{tr}+\partial_t\cancel{\Gamma}^\theta_{t\theta}+\partial_t\cancel{\Gamma}^\varphi_{t\varphi}$ substitution applied

(e) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{tt} = \Gamma^t_{tt}\Gamma^t_{tt}+\Gamma^r_{tr}\Gamma^t_{tt}+\Gamma^\theta_{t\theta}\Gamma^t_{tt}+\Gamma^\varphi_{t\varphi}\Gamma^t_{tt}$
$+\Gamma^t_{rt}\Gamma^r_{tt}+\Gamma^r_{rr}\Gamma^r_{tt}+\Gamma^\theta_{r\theta}\Gamma^r_{tt}+\Gamma^\varphi_{r\varphi}\Gamma^r_{tt}$
$+\Gamma^t_{\theta t}\Gamma^\theta_{tt}+\Gamma^r_{\theta r}\Gamma^\theta_{tt}+\Gamma^\theta_{\theta\theta}\Gamma^\theta_{tt}+\Gamma^\varphi_{\theta\varphi}\Gamma^\theta_{tt}$
$+\Gamma^t_{\varphi t}\Gamma^\varphi_{tt}+\Gamma^r_{\varphi r}\Gamma^\varphi_{tt}+\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{tt}+\Gamma^\varphi_{\varphi\varphi}\Gamma^\varphi_{tt}$

(f) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{tt} = \cancel{\Gamma}^t_{tt}\cancel{\Gamma}^t_{tt}+\cancel{\Gamma}^r_{tr}\cancel{\Gamma}^t_{tt}+\cancel{\Gamma}^\theta_{t\theta}\cancel{\Gamma}^t_{tt}+\cancel{\Gamma}^\varphi_{t\varphi}\cancel{\Gamma}^t_{tt}$
$+\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right)+\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r A}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right)$
$+\cancel{\Gamma}^t_{\theta t}\cancel{\Gamma}^\theta_{tt}+\cancel{\Gamma}^r_{\theta r}\cancel{\Gamma}^\theta_{tt}+\cancel{\Gamma}^\theta_{\theta\theta}\cancel{\Gamma}^\theta_{tt}+(\cot\theta)\cancel{\Gamma}^\theta_{tt}$
$+\cancel{\Gamma}^t_{\varphi t}\cancel{\Gamma}^\varphi_{tt}+\cancel{\Gamma}^r_{\varphi r}\cancel{\Gamma}^\varphi_{tt}+\cancel{\Gamma}^\theta_{\varphi\theta}\cancel{\Gamma}^\varphi_{tt}+\cancel{\Gamma}^\varphi_{\varphi\varphi}\cancel{\Gamma}^\varphi_{tt}$ substitution applied

(g) $\Gamma^\mu_{\sigma t}\Gamma^\sigma_{t\mu} = \Gamma^t_{tt}\Gamma^t_{tt}+\Gamma^r_{tt}\Gamma^t_{tr}+\Gamma^\theta_{tt}\Gamma^t_{t\theta}+\Gamma^\varphi_{tt}\Gamma^t_{t\varphi}$
$+\Gamma^t_{rt}\Gamma^r_{tt}+\Gamma^r_{rt}\Gamma^r_{tr}+\Gamma^\theta_{rt}\Gamma^r_{t\theta}+\Gamma^\varphi_{rt}\Gamma^r_{t\varphi}$
$+\Gamma^t_{\theta t}\Gamma^\theta_{tt}+\Gamma^r_{\theta t}\Gamma^\theta_{tr}+\Gamma^\theta_{\theta t}\Gamma^\theta_{t\theta}+\Gamma^\varphi_{\theta t}\Gamma^\theta_{t\varphi}$
$+\Gamma^t_{\varphi t}\Gamma^\varphi_{tt}+\Gamma^r_{\varphi t}\Gamma^\varphi_{tr}+\Gamma^\theta_{\varphi t}\Gamma^\varphi_{t\theta}+\Gamma^\varphi_{\varphi t}\Gamma^\varphi_{t\varphi}$

(h) $\Gamma^\mu_{\sigma t}\Gamma^\sigma_{t\mu} = \cancel{\Gamma}^t_{tt}\cancel{\Gamma}^t_{tt}+\left(\frac{\partial_r A}{2B}\right)\left(\frac{\partial_r A}{2A}\right)+\cancel{\Gamma}^\theta_{tt}\cancel{\Gamma}^t_{t\theta}+\cancel{\Gamma}^\varphi_{tt}\cancel{\Gamma}^t_{t\varphi}$
$+\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right)+\cancel{\Gamma}^r_{rt}\cancel{\Gamma}^r_{tr}+\cancel{\Gamma}^\theta_{rt}\cancel{\Gamma}^r_{t\theta}+\cancel{\Gamma}^\varphi_{rt}\cancel{\Gamma}^r_{t\varphi}$
$+\cancel{\Gamma}^t_{\theta t}\cancel{\Gamma}^\theta_{tt}+\cancel{\Gamma}^r_{\theta t}\cancel{\Gamma}^\theta_{tr}+\cancel{\Gamma}^\theta_{\theta t}\cancel{\Gamma}^\theta_{t\theta}+\cancel{\Gamma}^\varphi_{\theta t}\cancel{\Gamma}^\theta_{t\varphi}$
$+\cancel{\Gamma}^t_{\varphi t}\cancel{\Gamma}^\varphi_{tt}+\cancel{\Gamma}^r_{\varphi t}\cancel{\Gamma}^\varphi_{tr}+\cancel{\Gamma}^\theta_{\varphi t}\cancel{\Gamma}^\varphi_{t\theta}+\cancel{\Gamma}^\varphi_{\varphi t}\cancel{\Gamma}^\varphi_{t\varphi}$ substitution applied

(i)

$$R_{tt} = \partial_r\left(\frac{\partial_r A}{2B}\right)+ \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right)+\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r A}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r A}{2B}\right)- \left[\left(\frac{\partial_r A}{2B}\right)\left(\frac{\partial_r A}{2A}\right)+\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right)\right] \tag{7}$$

(j) $R_{tt} = \partial_r\frac{\partial_r A}{2B}+\left[\frac{\partial_r A}{2A}\frac{\partial_r A}{2B}+\frac{\partial_r B}{2B}\frac{\partial_r A}{2B}+\frac{\partial_r A}{2B}\frac{1}{r}+\frac{\partial_r A}{2B}\frac{1}{r}\right]-\left[\frac{\partial_r A}{2B}\left(\frac{\partial_r A}{2A}\right)-\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2B}\right)\right]$

(k) $R_{tt} = \partial_r\frac{\partial_r A}{2B}+\frac{\partial_r B}{2B}\frac{\partial_r A}{2B}+2\frac{\partial_r A}{2B}\frac{1}{r}+\frac{\partial_r A}{2B}\frac{\partial_r A}{2A}$

(l) $R_{tt} = \frac{1}{2B}\partial_r'' A+\frac{\partial_r A}{2}B^{-2}(-1)(\partial_r B)+\frac{\partial_r A\partial_r B}{4B^2}+\frac{\partial_r A}{Br}+\frac{(\partial_r A)^2}{4AB}$ - apply product rule on $\partial_r\frac{\partial_r A}{2B}$.

(m)

$$R_{tt} = 2ABr\partial_r'' A - Ar\partial_r A\partial_r B + 4BA\partial_r A + Br(\partial_r A)^2 \tag{8}$$

2. $R_{rr} = \partial_\mu\Gamma^\mu_{rr} - \partial_r\Gamma^\mu_{r\mu} + \Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{rr} - \Gamma^\mu_{\sigma r}\Gamma^\sigma_{r\mu}$

(a) $\partial_\mu\Gamma^\mu_{rr} = \partial_t\Gamma^t_{rr}+\partial_r\Gamma^r_{rr}+\partial_\theta\Gamma^\theta_{rr}+\partial_\varphi\Gamma^\varphi_{rr}$

(b) $\partial_\mu\Gamma^\mu_{rr}\partial_t\cancel{\Gamma}^t_{rr}+\partial_r\left(\frac{\partial_r B}{2B}\right)+\partial_\theta\cancel{\Gamma}^\theta_{rr}+\partial_\varphi\cancel{\Gamma}^\varphi_{rr}$ substitution applied

(c) $\partial_r\Gamma^\mu_{r\mu} = \partial_r\Gamma^t_{rt}+\partial_r\Gamma^r_{rr}+\partial_r\Gamma^\theta_{r\theta}+\partial_r\Gamma^\varphi_{r\varphi}$

(d) $\partial_r\Gamma^\mu_{r\mu} = \partial_r\left(\frac{\partial_r A}{2A}\right)+\partial_r\left(\frac{\partial_r B}{2B}\right)+\partial_r\left(\frac{1}{r}\right)+\partial_r\left(\frac{1}{r}\right)$ substitution applied

(e) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{rr} = \Gamma^t_{tt}\Gamma^t_{rr}+\Gamma^r_{tr}\Gamma^t_{rr}+\Gamma^\theta_{t\theta}\Gamma^t_{rr}+\Gamma^\varphi_{t\varphi}\Gamma^t_{rr}$
$+\Gamma^t_{rt}\Gamma^r_{rr}+\Gamma^r_{rr}\Gamma^r_{rr}+\Gamma^\theta_{r\theta}\Gamma^r_{rr}+\Gamma^\varphi_{r\varphi}\Gamma^r_{rr}$
$+\Gamma^t_{\theta t}\Gamma^\theta_{rr}+\Gamma^r_{\theta r}\Gamma^\theta_{rr}+\Gamma^\theta_{\theta\theta}\Gamma^\theta_{rr}+\Gamma^\varphi_{\theta\varphi}\Gamma^\theta_{rr}$
$+\Gamma^t_{\varphi t}\Gamma^\varphi_{rr}+\Gamma^r_{\varphi r}\Gamma^\varphi_{rr}+\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{rr}+\Gamma^\varphi_{\varphi\varphi}\Gamma^\varphi_{rr}$

(f) $\Gamma^{\mu}_{\sigma\mu}\Gamma^{\sigma}_{rr} = \cancel{\Gamma}^{t}_{tt}\cancel{\Gamma}^{t}_{rr}+\cancel{\Gamma}^{r}_{tr}\cancel{\Gamma}^{t}_{rr}+\cancel{\Gamma}^{\theta}_{t\theta}\cancel{\Gamma}^{t}_{rr}+\cancel{\Gamma}^{\varphi}_{t\varphi}\cancel{\Gamma}^{t}_{rr}$
$+\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r B}{2B}\right)$
$+\cancel{\Gamma}^{t}_{\theta t}\cancel{\Gamma}^{\theta}_{rr}+\cancel{\Gamma}^{r}_{\theta r}\cancel{\Gamma}^{\theta}_{rr}+\cancel{\Gamma}^{\theta}_{\theta\theta}\cancel{\Gamma}^{\theta}_{rr}+(\cot\theta)\cancel{\Gamma}^{\theta}_{rr}$
$+\cancel{\Gamma}^{t}_{\varphi t}\cancel{\Gamma}^{\varphi}_{rr}+\cancel{\Gamma}^{r}_{\varphi r}\cancel{\Gamma}^{\varphi}_{rr}+\cancel{\Gamma}^{\theta}_{\varphi\theta}\cancel{\Gamma}^{\varphi}_{rr}+\cancel{\Gamma}^{\varphi}_{\varphi\varphi}\cancel{\Gamma}^{\varphi}_{rr}$ substitution applied

(g) $\Gamma^{\mu}_{\sigma r}\Gamma^{\sigma}_{r\mu} = \Gamma^{t}_{tr}\Gamma^{t}_{rt}+\Gamma^{r}_{tr}\Gamma^{t}_{rr}+\Gamma^{\theta}_{tr}\Gamma^{t}_{r\theta}+\Gamma^{\varphi}_{tr}\Gamma^{t}_{r\varphi}$
$+\Gamma^{t}_{rr}\Gamma^{r}_{rt}+\Gamma^{r}_{rr}\Gamma^{r}_{rr}+\Gamma^{\theta}_{rr}\Gamma^{r}_{r\theta}+\Gamma^{\varphi}_{rr}\Gamma^{r}_{r\varphi}$
$+\Gamma^{t}_{\theta r}\Gamma^{\theta}_{rt}+\Gamma^{r}_{\theta r}\Gamma^{\theta}_{rr}+\Gamma^{\theta}_{\theta r}\Gamma^{\theta}_{r\theta}+\Gamma^{\varphi}_{\theta r}\Gamma^{\theta}_{r\varphi}$
$+\Gamma^{t}_{\varphi r}\Gamma^{\varphi}_{rt}+\Gamma^{r}_{\varphi r}\Gamma^{\varphi}_{rr}+\Gamma^{\theta}_{\varphi r}\Gamma^{\varphi}_{r\theta}+\Gamma^{\varphi}_{\varphi r}\Gamma^{\varphi}_{r\varphi}$

(h) $\Gamma^{\mu}_{\sigma r}\Gamma^{\sigma}_{r\mu} = \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2A}\right)+\cancel{\Gamma}^{r}_{tr}\cancel{\Gamma}^{t}_{rr}+\cancel{\Gamma}^{\theta}_{tr}\cancel{\Gamma}^{t}_{r\theta}+\cancel{\Gamma}^{\varphi}_{tr}\cancel{\Gamma}^{t}_{r\varphi}$
$+\cancel{\Gamma}^{t}_{rr}\cancel{\Gamma}^{r}_{rt}+\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r B}{2B}\right)+\cancel{\Gamma}^{\theta}_{rr}\cancel{\Gamma}^{r}_{r\theta}+\cancel{\Gamma}^{\varphi}_{rr}\cancel{\Gamma}^{r}_{r\varphi}$
$+\cancel{\Gamma}^{t}_{\theta r}\cancel{\Gamma}^{\theta}_{rt}+\cancel{\Gamma}^{r}_{\theta r}\cancel{\Gamma}^{\theta}_{rr}+\left(\frac{1}{r}\right)\left(\frac{1}{r}\right)+\cancel{\Gamma}^{\varphi}_{\theta r}\cancel{\Gamma}^{\theta}_{r\varphi}$
$+\cancel{\Gamma}^{t}_{\varphi r}\cancel{\Gamma}^{\varphi}_{rt}+\cancel{\Gamma}^{r}_{\varphi r}\cancel{\Gamma}^{\varphi}_{rr}+\cancel{\Gamma}^{\theta}_{\varphi r}\cancel{\Gamma}^{\varphi}_{r\theta}+\left(\frac{1}{r}\right)\left(\frac{1}{r}\right)$ substitution applied

(i)

$$\begin{aligned} R_{rr} = \partial_r\left(\frac{\partial_r B}{2B}\right) - \\ \left[\partial_r\left(\frac{\partial_r A}{2A}\right)+\partial_r\left(\frac{\partial_r B}{2B}\right)+\partial_r\left(\frac{1}{r}\right)+\partial_r\left(\frac{1}{r}\right)\right]+ \\ \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r B}{2B}\right) - \\ \left[\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2A}\right)+\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{1}{r}\right)+\left(\frac{1}{r}\right)\left(\frac{1}{r}\right)\right] \end{aligned} \tag{9}$$

(j)

$$\begin{aligned} R_{rr} = \cancel{\partial_r\left(\frac{\partial_r B}{2B}\right)} - \\ \left[\partial_r\left(\frac{\partial_r A}{2A}\right)+\cancel{\partial_r\left(\frac{\partial_r B}{2B}\right)}+\partial_r\left(\frac{1}{r}\right)+\partial_r\left(\frac{1}{r}\right)\right]+ \\ \left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r B}{2B}\right)+\cancel{\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r B}{2B}\right)}+\left(\frac{1}{r}\right)\left(\frac{\partial_r B}{2B}\right)+\left(\frac{1}{r}\right)\left(\frac{\partial_r B}{2B}\right) - \\ \left[\left(\frac{\partial_r A}{2A}\right)\left(\frac{\partial_r A}{2A}\right)+\cancel{\left(\frac{\partial_r B}{2B}\right)\left(\frac{\partial_r B}{2B}\right)}+\left(\frac{1}{r}\right)\left(\frac{1}{r}\right)+\left(\frac{1}{r}\right)\left(\frac{1}{r}\right)\right] \end{aligned}$$

(k) $R_{rr} = -\frac{\partial_r^{''}A}{2A}-\frac{\partial_r A}{2A^2}(-1)\partial_r A-2\left(\frac{1}{r^2}\right)(-1)+\frac{\partial_r A\partial_r B}{4AB}-\frac{\partial_r B}{rB}-\frac{(\partial_r A)^2}{4A^2}+2\left(\frac{1}{r^2}\right) =$ - using product rule and power rule for differentials involved.

(l)

$$R_{rr} = -2ArB\partial_r^{''}A - rB(\partial_r A)^2 + rA\partial_r A\partial_r B + 4A^2\partial_r B \tag{10}$$

3. $R_{\theta\theta} = \partial_\mu\Gamma^{\mu}_{\theta\theta} - \partial_\theta\Gamma^{\mu}_{\theta\mu} + \Gamma^{\mu}_{\sigma\mu}\Gamma^{\sigma}_{\theta\theta} - \Gamma^{\mu}_{\sigma\theta}\Gamma^{\sigma}_{\theta\mu}$

(a) $\partial_\mu\Gamma^{\mu}_{\theta\theta} = \partial_t\Gamma^{t}_{\theta\theta}+\partial_r\Gamma^{r}_{\theta\theta}+\partial_\theta\Gamma^{\theta}_{\theta\theta}+\partial_\varphi\Gamma^{\varphi}_{\theta\theta}$

(b) $\partial_\mu\Gamma^{\mu}_{\theta\theta} = \partial_t\cancel{\Gamma}^{t}_{\theta\theta}+\partial_r\left(\frac{-r}{B}\right)+\partial_\theta\cancel{\Gamma}^{\theta}_{\theta\theta}+\partial_\varphi\cancel{\Gamma}^{\varphi}_{\theta\theta}$ substitution applied

(c) $\partial_\theta\Gamma^{\mu}_{\theta\mu} = \partial_\theta\Gamma^{t}_{\theta t}+\partial_\theta\Gamma^{r}_{\theta r}+\partial_\theta\Gamma^{\theta}_{\theta\theta}+\partial_\theta\Gamma^{\varphi}_{\theta\varphi}$

(d) $\partial_\theta\Gamma^{\mu}_{\theta\mu} = \partial_\theta\cancel{\Gamma}^{t}_{\theta t}+\partial_\theta\cancel{\Gamma}^{r}_{\theta r}+\partial_\theta\cancel{\Gamma}^{\theta}_{\theta\theta}+\partial_\theta(\cot\theta)$ substitution applied

(e) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{\theta\theta} = \Gamma^t_{tt}\Gamma^t_{\theta\theta}+\Gamma^r_{tr}\Gamma^t_{\theta\theta}+\Gamma^\theta_{t\theta}\Gamma^t_{\theta\theta}+\Gamma^\varphi_{t\varphi}\Gamma^t_{\theta\theta}$
$+\Gamma^t_{rt}\Gamma^r_{\theta\theta}+\Gamma^r_{rr}\Gamma^r_{\theta\theta}+\Gamma^\theta_{r\theta}\Gamma^r_{\theta\theta}+\Gamma^\varphi_{r\varphi}\Gamma^r_{\theta\theta}$
$+\Gamma^t_{\theta t}\Gamma^\theta_{\theta\theta}+\Gamma^r_{\theta r}\Gamma^\theta_{\theta\theta}+\Gamma^\theta_{\theta\theta}\Gamma^\theta_{\theta\theta}+\Gamma^\varphi_{\theta\varphi}\Gamma^\theta_{\theta\theta}$
$+\Gamma^t_{\varphi t}\Gamma^\varphi_{\theta\theta}+\Gamma^r_{\varphi r}\Gamma^\varphi_{\theta\theta}+\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{\theta\theta}+\Gamma^\varphi_{\varphi\varphi}\Gamma^\varphi_{\theta\theta}$

(f) $\Gamma^\mu_{\sigma\mu}\Gamma^\sigma_{\theta\theta} = \cancel{\Gamma^t_{tt}\Gamma^t_{\theta\theta}}+\cancel{\Gamma^r_{tr}\Gamma^t_{\theta\theta}}+\cancel{\Gamma^\theta_{t\theta}\Gamma^t_{\theta\theta}}+\cancel{\Gamma^\varphi_{t\varphi}\Gamma^t_{\theta\theta}}$
$+\left(\frac{\partial_r A}{2A}\right)\left(\frac{-r}{B}\right)+\left(-\frac{\partial_r B}{2B}\right)\left(\frac{-r}{B}\right)+\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)+\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)$
$+\cancel{\Gamma^t_{\theta t}\Gamma^\theta_{\theta\theta}}+\cancel{\Gamma^r_{\theta r}\Gamma^\theta_{\theta\theta}}+\cancel{\Gamma^\theta_{\theta\theta}\Gamma^\theta_{\theta\theta}}+(\cot\theta)\cancel{\Gamma^\theta_{\theta\theta}}$
$+\cancel{\Gamma^t_{\varphi t}\Gamma^\varphi_{\theta\theta}}+\cancel{\Gamma^r_{\varphi r}\Gamma^\varphi_{\theta\theta}}+\cancel{\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{\theta\theta}}+\cancel{\Gamma^\varphi_{\varphi\varphi}\Gamma^\varphi_{\theta\theta}}$ substitution applied

(g) $\Gamma^\mu_{\sigma\theta}\Gamma^\sigma_{\theta\mu} = \Gamma^t_{t\theta}\Gamma^t_{\theta t}+\Gamma^r_{t\theta}\Gamma^t_{\theta r}+\Gamma^\theta_{t\theta}\Gamma^t_{\theta\theta}+\Gamma^\varphi_{t\theta}\Gamma^t_{\theta\varphi}$
$+\Gamma^t_{r\theta}\Gamma^r_{\theta t}+\Gamma^r_{r\theta}\Gamma^r_{\theta r}+\Gamma^\theta_{r\theta}\Gamma^r_{\theta\theta}+\Gamma^\varphi_{r\theta}\Gamma^r_{\theta\varphi}$
$+\Gamma^t_{\theta\theta}\Gamma^\theta_{\theta t}+\Gamma^r_{\theta\theta}\Gamma^\theta_{\theta r}+\Gamma^\theta_{\theta\theta}\Gamma^\theta_{\theta\theta}+\Gamma^\varphi_{\theta\theta}\Gamma^\theta_{\theta\varphi}$
$+\Gamma^t_{\varphi\theta}\Gamma^\varphi_{\theta t}+\Gamma^r_{\varphi\theta}\Gamma^\varphi_{\theta r}+\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{\theta\theta}+\Gamma^\varphi_{\varphi\theta}\Gamma^\varphi_{\theta\varphi}$

(h) $\Gamma^\mu_{\sigma\theta}\Gamma^\sigma_{\theta\mu} = \cancel{\Gamma^t_{t\theta}\Gamma^t_{\theta t}}+\cancel{\Gamma^r_{t\theta}\Gamma^t_{\theta r}}+\cancel{\Gamma^\theta_{t\theta}\Gamma^t_{\theta\theta}}+\cancel{\Gamma^\varphi_{t\theta}\Gamma^t_{\theta\varphi}}$
$+\cancel{\Gamma^t_{r\theta}\Gamma^r_{\theta t}}+\cancel{\Gamma^r_{r\theta}\Gamma^r_{\theta r}}+\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)+\cancel{\Gamma^\varphi_{r\theta}\Gamma^r_{\theta\varphi}}$
$+\cancel{\Gamma^t_{\theta\theta}\Gamma^\theta_{\theta t}}+\left(\frac{-r}{B}\right)\left(\frac{1}{r}\right)+\cancel{\Gamma^\theta_{\theta\theta}\Gamma^\theta_{\theta\theta}}+\cancel{\Gamma^\varphi_{\theta\theta}\Gamma^\theta_{\theta\varphi}}$
$+\cancel{\Gamma^t_{\varphi\theta}\Gamma^\varphi_{\theta t}}+\cancel{\Gamma^r_{\varphi\theta}\Gamma^\varphi_{\theta r}}+\cancel{\Gamma^\theta_{\varphi\theta}\Gamma^\varphi_{\theta\theta}}+(\cot\theta)\;(\cot\theta)$ substitution applied

$$\begin{aligned} R_{\theta\theta} = \partial_r \left(\frac{-r}{B}\right) - \\ \partial_\theta\,(\cot\theta) + \\ \left[\left(\frac{\partial_r A}{2A}\right)\left(\frac{-r}{B}\right) + \left(-\frac{\partial_r B}{2B}\right)\left(\frac{-r}{B}\right) + \left(\frac{1}{r}\right)\left(\frac{-r}{B}\right)\right] - \\ \left[\left(\frac{1}{r}\right)\left(\frac{-r}{B}\right) + \left(\frac{-r}{B}\right)\left(\frac{1}{r}\right) + (\cot\theta)\,(\cot\theta)\right] \end{aligned} \tag{11}$$

(i) $R_{\theta\theta} = \partial_r\frac{-r}{B} - \partial_\theta\cot\theta + [(\frac{\partial_r A}{2A})(\frac{-r}{B}) + (-\frac{\partial_r B}{2B})(\frac{-r}{B}) + (\frac{1}{r})(\frac{-r}{B}) + (\frac{1}{r})(\frac{-r}{B})] - [(\frac{1}{r})(\frac{-r}{B}) + (\frac{-r}{B})(\frac{1}{r}) + (\cot\theta)(\cot\theta)]$

i. $\partial_r\cot\theta = -\frac{1}{sin^2\theta}$

ii. $\cot\theta\cot\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{1-sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} - 1$

iii. $\partial_r\frac{-r}{B} = \frac{-1}{B} + r\frac{-1}{B^2}\partial_r B$

(j)

$$R_{\theta\theta} = \frac{-1}{B} + 1 + \frac{r\partial_r A}{2AB} - \frac{r\partial_r B}{2B^2} = -2AB + 2AB^2 - rB\partial_r A + rA\partial_r B \tag{12}$$

4. Since $R_{tt} = R_{rr} = R_{\theta\theta} = 0$ we can solve this system of equations by adding the first eq 8 and eq 10 and dividing by $4A$. We are left with $B\partial_r A + A\partial_r B = 0$ (see below, rearranged slightly)

(a)

$$R_{tt} = 2ABr\partial_r^{''}A - Ar\partial_r A\partial_r B + Br(\partial_r A)^2 + 4BA\partial_r A = 0$$
$$R_{rr} = -2ArB\partial_r^{''}A + rA\partial_r A\partial_r B - rB(\partial_r A)^2 + 4A^2\partial_r B = 0$$

(b) $4BA\partial_r A + 4A^2\partial_r B = 0$

(c) $B\partial_r A + A\partial_r B = 0$

5. $B\partial_r A + A\partial_r B = \partial_r(AB) = 0$ which implies $AB = K$ where $K = const.$